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ABSTRACT

The stability of single mode operation of FEL oscillators is investigated. We consider two models of an untapered FEL oscillator. The first model is called the klystron model. In it the FEL interaction occurs at two points: a prebunching point and an energy extraction point. In this model the nonlinear electron dynamics are solvable exactly leading to a complex delay equation for the wave fields. The stability of single mode operation can then be determined easily as a function of single pass gain, energy mismatch-frequency, and the difference between the group velocity of the radiation and the beam velocity. The second, more realistic model has a distributed interaction region of finite length. Stability of single mode operation in this device must be determined numerically. The results of the models in low gain regime will be compared and the parameter regimes where stable-single mode operation is possible will be determined.

I. INTRODUCTION

Many applications of free electron lasers require a coherent single frequency output. However, the FEL interaction region is large and in oscillators many modes of the interaction cavity are linearly unstable. Therefore it is desirable to know under what circumstances nonlinear operation in a single frequency mode will occur. This problem has been considered previously by a number of authors.[1,2,3] Our work closely parallels that of Refs. (1) and (2). Specifically, we consider two cases in which reduction in computation is achieved by considering a model system or by exploiting a small parameter.

We begin with the one pendulum equation describing the space and time evolution of the particle energy and phase and signal field in an FEL,

$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z}\right)\psi = - \delta\gamma(\omega v_z) \frac{\partial}{\partial \gamma_r} (v_z^{-1}) \, , \tag{1}$$

$$\left(\frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z}\right)(\delta\gamma + \gamma_r) = -\frac{\partial}{\partial \psi}\left\{\omega K\left[\frac{qA}{mc^2} e^{i\psi} + \frac{qA^*}{mc^2} e^{-i\psi}\right]\right\} , \qquad (2)$$

$$2i\left(\frac{\omega}{c^2}\frac{\partial}{\partial t} + k_r \frac{\partial}{\partial z}\right)\frac{qA}{mc^2} = \frac{4\pi q}{mc^2}\frac{1}{v_z} K \langle e^{-i\psi}\rangle , \qquad (3)$$

where ψ and $\delta\gamma$ are a particle phase and energy mismatch, and $A(z,t)$ is the complex amplitude of the signal field. The definition of the other quantities appearing in Eqs. (1)-(3) follows: ω and k_z are the wave frequency and wavenumber, j is an effective current density, $v_z(\gamma_r)$ is the mean beam speed, for the case considered here γ_r is a constant and gives the energy of a particle exactly resonant with the carrier signal. The undulator parameter is defined by

$$K = \frac{qA_w(z)}{mc^2\gamma_r} .$$

Since we are considering oscillators it is necessary to augment the wave equation (3) with a boundary condition describing the return path of the radiation,

$$A(z=0, t) = RA(z=L, t-L/v_g) , \qquad (4)$$

where R is the combined reflection coefficient for the two mirrors, L is the separation between mirrors and $v_g = k_r c^2/\omega$ is the group velocity of the radiation. (Note that R is a complex number.)

We now make two transformations to Eqs. (1)-(3). The first is simply to normalize all the variables. Specifically we introduce,

$$p = \delta\gamma \frac{L\omega}{c(\gamma_r^2 - 1)^{3/2}} , \qquad (5a)$$

$$\hat{a} = \left(\frac{qA}{mc^2}\right)\frac{(\omega L/c)^2 K}{\beta_z(\gamma_r^2 - 1)^{3/2}} , \qquad (5b)$$

and

$$\hat{\xi} = z/L \qquad \hat{\tau} = v_g t/L . \qquad (5c)$$

Second, we define a new coordinate system moving with the mean beam speed

$$\xi' = \xi \qquad (6a)$$

$$\tau' = \tau - \beta_* \xi , \qquad (6b)$$

where $\beta_* = v_g/v_z$ is the ratio of the radiation group velocity to the mean beam speed. With these changes the governing equations become,

$$\frac{\partial \psi}{\partial \xi'} = p \ , \tag{7}$$

$$\frac{\partial p}{\partial \xi'} = -\frac{1}{2} X(\xi')(ae^{i\psi} - a^* e^{-i\psi}) \ , \tag{8}$$

$$2i\left[(1-\beta_*) \frac{\partial}{\partial \tau'} + \frac{\partial}{\partial \xi'}\right] a = X(\xi')\hat{I}\langle e^{-i\psi}\rangle \ , \tag{9}$$

with the boundary condition

$$a(0,\tau') = Ra\big(1, \tau' - (1+\beta_*)\big) \ . \tag{10}$$

The function $X(\xi)=A_w(z)/A_{w0}$ gives the profile of the coupling coefficient between particles and fields. We note that the system of equations (7)–(10) has four dimensionless parameters: the normalized beam current

$$\hat{I} = \frac{4\pi j}{I_A} \frac{L^3 \omega}{c} \frac{K^2}{\beta_z^2 (\gamma_r^2 - 1)^{3/2}} \ ,$$

where $I_A = mc^3/q = 1.7 \times 10^4$ Amps, the relative velocities β_*, the reflection coefficient R, and the injection momentum $p(\xi'=0)$.

In the remainder of this paper we will consider two ways this system can be further simplified.

II. KLYSTRON MODEL

In the klystron model[2,4] one assumes a specific form for the profile of the coupling coefficient,

$$X(\xi) = \delta(\xi') + \delta(\xi' - 1) \ .$$

Physically, this models an FEL with two short interaction regions separated by a drift space. In the first interaction region, $\xi'=0$, particles receive a kick in energy (p) depending on their entrance phase and the strength of the signal field $a(\xi=0,\tau')$. The particles then bunch in phase as they traverse the drift space producing a coherent current source which amplifies the radiation at the second interaction region. The equations of motion describing this process can be solved exactly yielding an expression for the output radiation field for a given input field. Inserting this expression in the feedback boundary condition produces the following two time delay equations,

$$a(0,\tau') = R\left\{ a(0, \tau'-2) + i \frac{\hat{I}}{2} e^{-ip(0)} a\big(0, \tau' - (1+\beta_*)\big)\hat{\eta} \right\} \ , \tag{11}$$

where

$$\hat{\eta} = \frac{J_1\big[|a(0, \tau' - (1+\beta_*))| \big]}{|a(0, \tau' - (1+\beta_*))|} \ ,$$

with J_1 a Bessel function.

Physically, the first term on the right-hand side of Eq. (11) represents the decay of the circulatng radiation which in our variables takes two units of time to complete one circuit of the resonator. The second term describes the amplification of the radiation due to the beam. The delay in this case is $(1+\beta_*)$, the "one" coming from the return trip of the radiation and the "β_*" coming from the time the particles take to traverse the drift region.

For equal beam and radiation speeds $(\beta_*=1)$ Eq. (11) becomes a simple return map in which each time slice of the radiation field evolves independently of its neighbor depending only on the value of the radiation field two units of time in the past. This would also be the case if we had set $\beta_*=1$ in Eq. (9) without introducing the klystron model. The function $\hat{\eta}(|a|)$ in this case would be replaced by some other transcendental function. However, as shown by Fiegenbaum[5] the behavior of all such return maps with quadratic maxima is universal. In particular, depending on beam current the sequence of field values $a(0,\tau-2n)$ may converge to a single value (single mode operation), or a periodic sequence, or for sufficiently high current a chaotic sequence. We note that the return map model is only appropriate for beam currents low enough to produce a sequence of constant values for $a(0,\tau)$. This is because even if the sequence converges to a period two or higher orbit the function $a(0,\tau)$ must necessarily have a discontinuity. In which case, no matter how small $1-\beta_*$ is since it multiplies the time derivative of $a(0,\tau)$ in Eq. (9) it can not be neglected. Thus, the two time delay, Eq. (11), is useful in that it allows for an assessment of the effects of finite slippage, i.e., $1-\beta_*\neq0$.

The stability of a single frequency nonlinear mode is determined as follows. We write the field in terms of an equilibrium value and a perturbation

$$a(\tau) = \exp(-i\omega_0\tau)[a_0 + \delta a(\tau)]$$

where a_0 is the constant amplitude of the single mode with frequency ω_0, and $\delta a(\tau) \sim \exp(-i\omega\tau)$ is the perturbation. The frequency and amplitude in equilibrium satisfy

$$1 = R\left[e^{2i\omega_0} + \frac{\hat{I}}{2} e^{i\hat{\theta}_0} \hat{\eta}(|a_0|) \right] , \qquad (12)$$

where $\theta_0=(\pi/2)+(1+\beta_*)\omega_0-p(0)$. Thus, for a given current $\hat{I}$, reflectivity

R, and injection momentum $p(0)$, Eq. (12) is a transcendental relation determining the field amplitude a_0 and frequency ω_0. The equilibrium relation is readily solved in the low gain regime where both I and $\nu=1-R$ are small. One finds $\omega_0 = n\pi + \delta\omega_0$ with

$$\frac{2\nu}{I} = \cos\theta_0 \hat{\eta}(|a_0|) \text{ and } \delta\omega_0 = -\frac{I}{4} \sin\theta_0 \hat{\eta}(|a_0|) .$$

Thus, the frequency is essentially that of a cavity mode with a small shift proportional to the beam current, and the field amplitude is determined by the gain and the ratio of the transmission coefficient (ν) to the current.

Linearizing the delay equation results in the following dispersion relation for the frequency ω of the perturbation

$$\{(e^{-2i\omega}-1) - Z_R[(1+g)e^{ik}-1]\} \times \{(e^{-2i\omega}-1) - Z_R[e^{ik}-1]\}$$

$$\text{(13)}$$

$$+ Z_I^2[(1+g)e^{ik}-1](e^{ik}-1) = 0 ,$$

where $k=(\hat{\beta}^*-1)\omega$, $g=a_0/\hat{\eta}(a_0)\partial\hat{\eta}/\partial a_0$, and $Z=1/2(\hat{\eta}\hat{R}Ie^{i\theta_0})$, while Z_R and Z_I are the real and imaginary part of Z, respectively.

In the low gain regime $|Z|\ll 1$ to lowest order Eq. (13) gives

$$(e^{-2i\omega} - 1) = 0$$

and the frequency of the perturbation is also approximately that of a cavity mode

$$\omega \tilde{=} \pi n \qquad (n \neq 0) .$$

The growth or damping of the mode is determined in next order by the equation

$$\hat{w}^2 - \cos\theta_0 \hat{w}[(2+g)e^{ik_n} - 2] + (e^{ik_n} - 1)[(1+g)e^{ik_n} - 1] = 0 ,$$

where $\hat{w}=(e^{-2i\omega}-1)/|Z|$ with $k_n=n\pi(\hat{\beta}^*-1)$. Regions of stability in the k vs. $\cos\theta_0$ plane are shown in Figs. (1) for different values of g. For the equilibrium the average change in momentum Δp is given in the low gain regime by,

$$\Delta p = - \cos\theta_0 |a_0|^2 \hat{\eta}(|a_0|) .$$

Thus, the value of $\cos\theta_0$ determines the dependence of the efficiency on the injection momentum. The quantity g depends only on a_0 and is related to the derivative of the efficiency with respect to the field amplitude,

$$\frac{|a_0|}{\Delta p} \frac{\partial(\Delta p)}{\partial|a_0|} = (2 + g) . \qquad \text{(14)}$$

The efficiency maximizes for $g=-2$ and $\cos\theta_0=1$.

The plots in Fig. (1a) are periodic in k with period 2π (only the fundamental period is shown). Thus, for an irrational value of (β_*-1), $k_n\bmod(2\pi)$ will eventually assume any value. In the klystron model, therefore, only equilibria with efficiency sufficiently maximized with respect to $\cos\theta_0$ (injection momenta) will be stable. Periodicity with respect to k is an artifact of the klystron model, and we may not expect this result to hold more generally.

A second more important restriction is noted by examining Fig. (1b) where $g\leq-2$, corresponding to a field amplitude in equilibrium that exceeds the value which maximizes efficiency. There it is seen that no single frequency equilibria are stable.

The preceeding discussion applied in the low gain regime where both the beam current $\hat{I}$ and transmission coefficient $\nu=(1-R)$ are small. In this case the stability boundaries depend only on the ratio $\hat{I}/\nu$ through the field amplitude a_0.

In the high gain regime an additional parameter measuring the current is introduced, $|Z|=(1/2)R\hat{I}\eta$. Stability boundaries in the $\cos\theta_0 - |Z|$ plane are shown in Fig. (2) for the high gain case. The large triangle in the upper left is the region of parameter space allowed by the equilibrium condition. The low gain results correspond to the region of the plane with small $|Z|$. As can be seen in Fig. (2) where $g>-2$ the stable operating regime occupies the small triangle in the upper left. The stable range of $\cos\theta_0$ values decreases as current is raised. Above a critical current no single mode operation is possible. For $g<-2$ we again find no regime where stable operation is possible.

III. MULTIMODE SIMULATIONS

We now drop the klystron model and return to the more realistic case of a constant coupling coefficient in the interaction region,

$$X(\xi) = \begin{cases} 1 \text{ for } 0 \leq \xi \leq 1 \\ 0 \text{ otherwise} \end{cases} .$$

To reduce the number of parameters we again assume that the beam current and transmission coefficient are small and we are in the low gain regime. Solution of the wave equation in the limit $\hat{I}\to0$ will reproduce the vacuum

cavity modes. Including the current and transmission losses will cause the complex amplitude of each mode to evolve slowly in time. To treat this problem mathematically we introduce a two time scale formulation writing the field amplitude

$$a(\xi,\tau) \simeq \sum_n a_n(\tau) e^{i\omega_n[(1-\beta_*)\xi - \tau_0]} , \qquad (15)$$

where τ_0 is the fast time associated with the time of flight of radiation through the cavity, $\omega_n = \pi n$ is the frequency, and τ is the slow time associated with the growth or decay of the modes.

The computational savings is achieved because the field is nearly periodic with period $\tau_0 = 2$. Thus, an ensemble of particles can be launched with τ_0 in the range $0 \leq \tau_0 \leq 2$, the currents computed, and the wave fields advanced by a time τ of the order of the growth time. This reduction has been achieved similarly in other FEL[1] and gyrotron simulations.[6-7]

The governing system of equations is found to be

$$\frac{\partial p}{\partial \xi} = \text{Im}(\sum_n a_n e^{i\psi_n(\xi,\tau_0)}) , \qquad (16)$$

$$\frac{\partial \psi}{\partial \xi} = p , \qquad (17)$$

where $\psi_n = \psi + \omega_n[(1-\beta_*)\xi - \tau_0]$ with initial conditions $\tau_0 \in [0,2]$, $\psi(0) \in [0,2\pi]$, and $p(0) = p_{inj}$ for the particles. The wave amplitudes are advanced on the time scale τ_s

$$\frac{\partial a_n}{\partial \tau_s} + \frac{1}{2} a_n = - \frac{1}{4} (\frac{\hat{I}}{\nu}) \langle\langle e^{-i\psi_n} \rangle\rangle \qquad (18)$$

where

$$\tau_s = \tau\nu,$$

and the double average is given by

$$\langle\langle ... \rangle\rangle = \int_0^{2\pi} \frac{d\psi(0)}{2\pi} \int_0^2 \frac{d\tau_0}{2} \int_0^1 d\xi (...) .$$

A sample run is shown in Fig. (3) where the magnitude of the amplitudes of the modes are shown at several times. The injection momentum and current to damping ratio were selected so that in the case of single mode operation the $n=-3$ mode would be the most efficient. At early times one sees that the $n=1$ mode grows fastest which is consistent with small

signal theory. As time goes by the system nonlinearly evolves toward a single mode (the n=0 mode). At the time of writing of this paper the code was still running so we can not report whether a final single mode solution is reached. However, we hope to examine parameter space and determine the regimes of single mode operation in the future.

IV. CONCLUSIONS

We have outlined two models capable of describing multimode effects in FEL oscillators. In the klystron model the wave field satisfies a two time delay equation. In the low gain regime single mode operation is achieved only at a field value less than that which maximizes the efficiency and then only for modes for which the efficiency has been nearly maximized with respect to mismatch. In the high gain regime the band of stable mismatch frequencies decreases as current increases. In the more realistic distributed interaction model one must resort to numerical simulation. However, the computation time and parameter space can be greatly reduced in the low gain regime.

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FIGURE CAPTIONS

Fig. 1. Stability regions in the low gain regime in $\cos\theta_0$ versus $k = n\pi(\beta^*-1)$ plane: (a) $g = -1.95$, (b) $g = -2.01$.

Fig. 2. Stability regions in the high gain regime in $\cos\theta_0$ versus $Z = 1/2(\hat{I}\hat{R}n)$ plane.

Fig. 3. Sample of the multimode simulation with 21 modes, $\beta^* = 1.2$, $(\hat{I}/\nu) = 112.74$, and $P_{inj} = 3.258$; (a) $\tau_s = 1.2$; (b) $\tau_s = 4.8$; (c) $\tau_s = 18$; (d) $\tau_s = 74.4$.
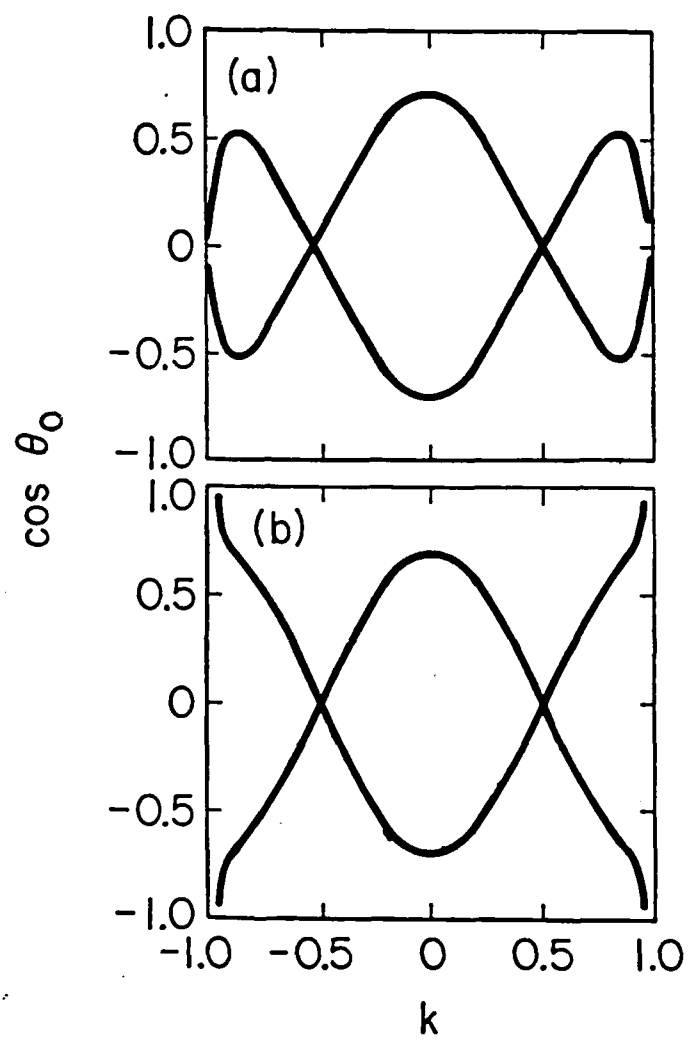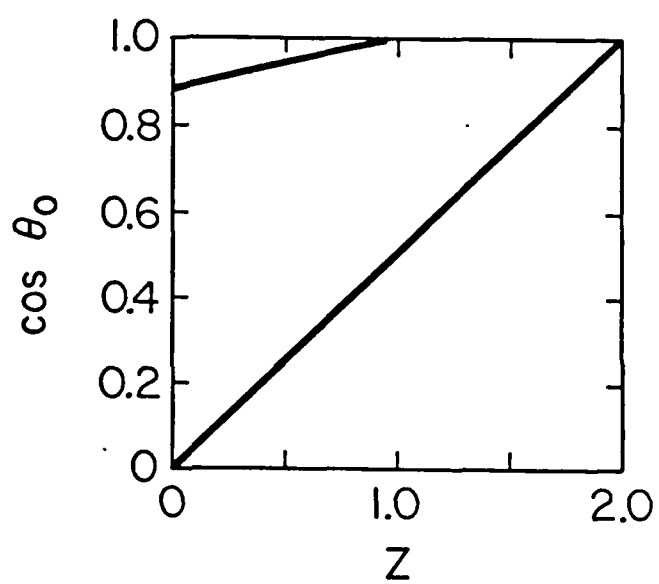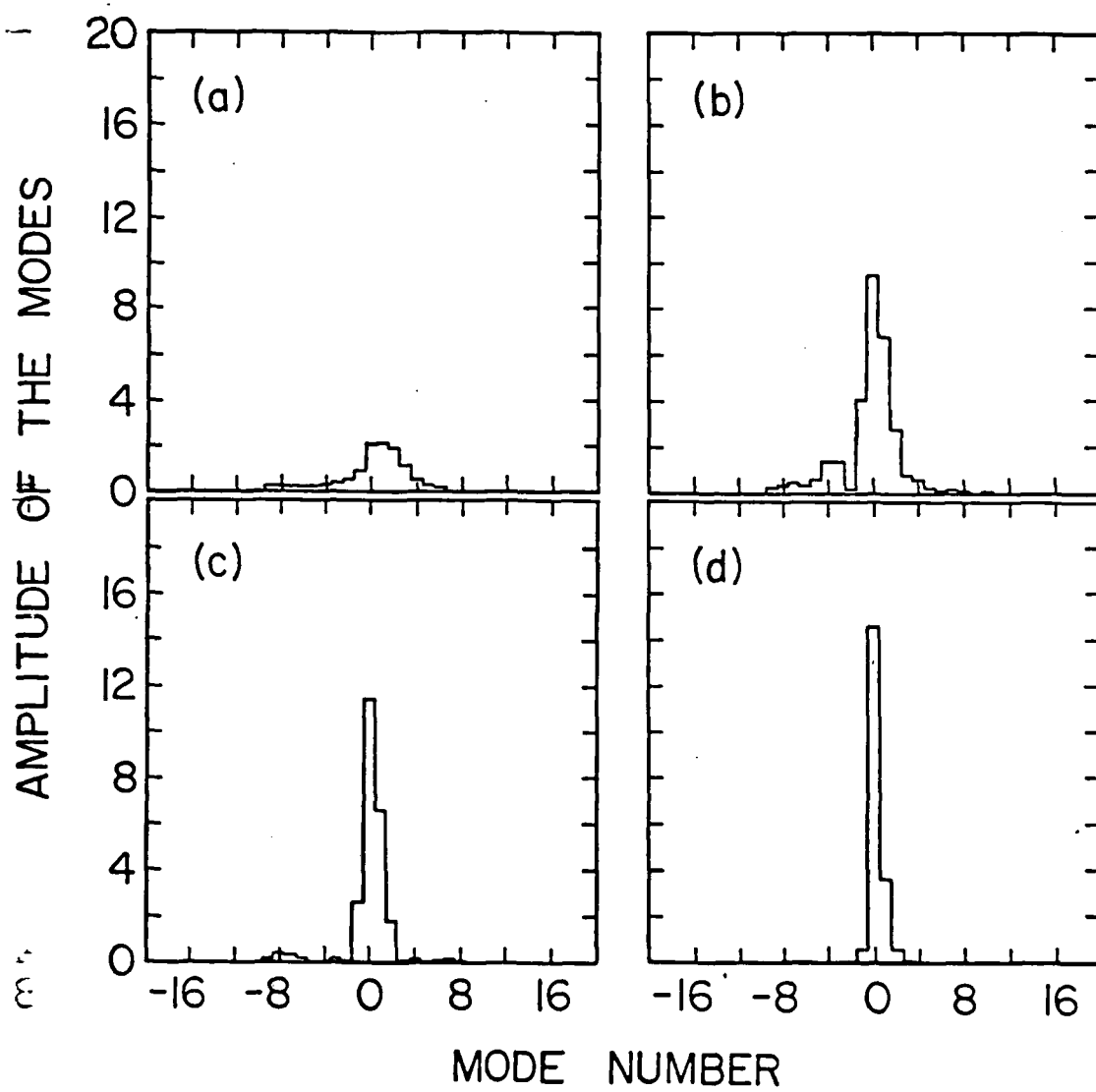
Fig. 1

Fig. 2

Fig. 3

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